Tetradec

Using Iannis Xenakis' methods of working with mathematical 'sieves' as its launching point, *Tetradec* uses pitch scales of increasing size to generate melodic material. A 'sieve', though not dissimilar to a *scale* in application, takes into account a base value (i.e. base-12 in 12-tone equal temperament scales), and considers the means by which a selection of values (the values to be let through the 'sieve') can be formulated. A harmonic minor scale, for example, contains values 0, 2, 3, 5, 7, 8 and 11 of the chromatic scale. The 'base' of the chromatic scale is here referred to as the *modulus* [*M*], and identification of values takes the format (*M*, *I*), where *I* is a value less than *M*. Values are said to be equivalent modulo when they give the same remainder when divided by *M*. For (12, 2), for example, 14, 26 and 38 are considered equivalent modulo, since they give a remainder of two when divided by 12—familiar thinking to the musician used to the concept of octave equivalence.[1]

A whole-tone scale might be constructed by taking the module (2,0), which is to say, 'start at pitch zero and take every second note'. Since diatonic scales do not proceed in equal steps, their construction requires taking the intersection of two or more modules. An *M* value must first be reduced to its product of prime factors to achieve this. The prime factorisation of 12, for example, is 2^2+3, which can be represented visually on a four by three matrix:

[1] Dimitrios Exarchos, "Iannis Xenakis and Sieve Theory: An Analysis of the Late Music (1984-1993)" (Goldsmiths, University of London, 2007), pp. 56–57.

		M=4			
		0	**1**	**2**	**3**
M=3	**0**	0	9	6	3
	1	4	1	10	7
	2	8	5	2	11

The grid is populated with the twelve values of the chromatic scale beginning with upper leftmost position and proceeding diagonally downwards. Taking the ‘coordinates’ of the desired values is an efficient method of determining the intersections required to produce a sieve. The harmonic-minor values provided above, for example, require taking the following set of intersections:[2]

$$(3,0)\cdot(4,0)+(3,2)\cdot(4,2)+(3,0)\cdot(4,3)+(3,2)\cdot(4,1)+(3,1)\cdot(4,3)+(3,2)\cdot(4,0)+(3,2)\cdot(4,3)$$

This formula suggests a number of transformations. So far, the *modulo*, 12, has corresponded to the number of values in the 12-tone tone chromatic scale. By taking values greater than 12, scales are produced which do not repeat at the octave. This was the genesis for the piece: by combining scales of increasing *M* values, new pitches are introduced, and the superimposition of these scales yields non-tonal sonorities. Further, sieves can be applied rhythmically, by taking a unit of time with *M* divisions. The piece was to explore superimpositions varying sieves between instruments.

[2] The symbol · here denotes the intersecting point of two modules, not the sum, as is the conventional use of the symbol.

After some experimentation, I found that the scale informally known as the Neapolitan scale, built on G to accommodate my chosen instruments' lowest notes, yielded the sound world I had in mind:

The scale has a number of notable features, perhaps most significantly the cluster of semitones about the root. As a sieve, the scale is notated thus:

$$(4,0)\cdot(3,0)+(4,1)\cdot(3,1)+(4,3)\cdot(3,0)+(4,1)\cdot(3,2)+(4,3)\cdot(3,1)+(4,0)\cdot(3,2)+(4,3)\cdot(3,2)$$

The piece begins with the superimposition of this and two transformations: an *M*=15 [0,1,3,5, 8,11,13] iteration, and an *M*=24 [0,1,3,8,11,17,19] iteration. In the introduction, violin 1 is assigned *M* values of 12 and 24, whilst violin 2 plays 12 and 15. Pitch and dynamic material are generated through the superimposition of sieves using the method shown in example 1, but pitch material is confined to *M*=12, rising the range of one octave over the course of the section. The cycle of 21 notes against 12 takes 44 bars to complete, and the cycle of 15 against 12, 21 bars to complete, which suggested a duration for the section. The sonorities generated by this simple process demonstrate the speed with which transformations of sieves can yield unexpected results, and contained in the introduction are many of the rhythmic elements of the later material.

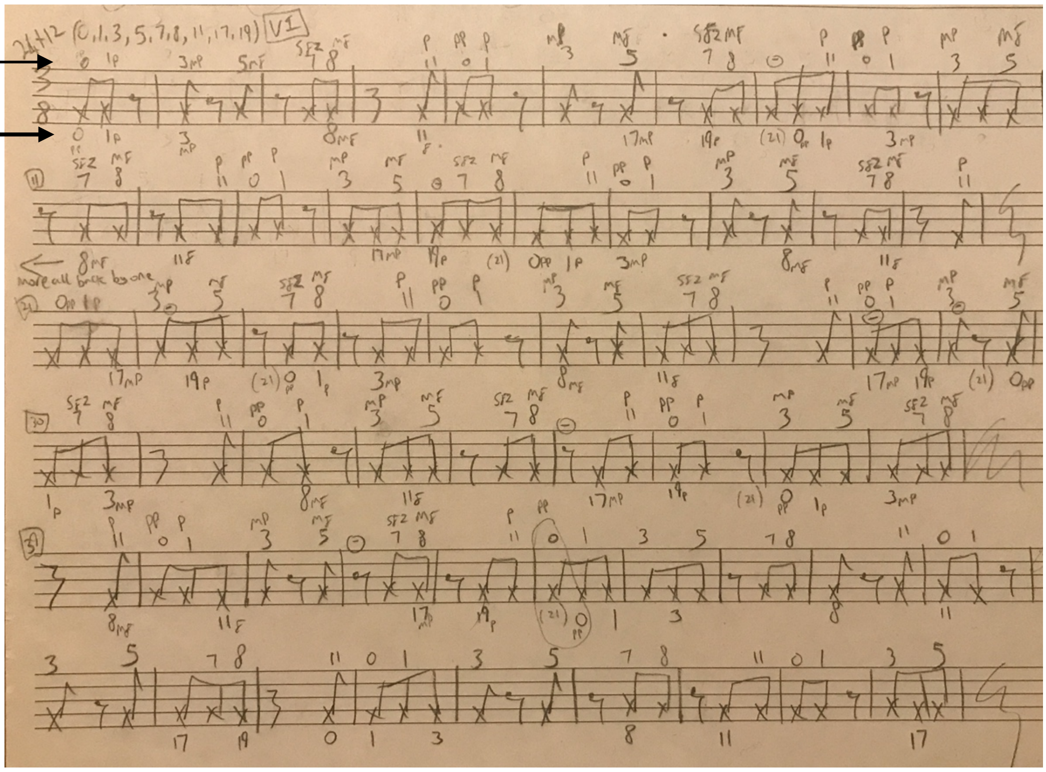
B 99–110 contain a transitionary passage made up ascending sieves. 99–102 contain *M*=in rhythmic unison. *M*=12 is then combined with an iteration of itself with doubled note values. Throughout this passage, violin 2 begins to take on its

Example 1: Method used to generate dynamic and rhythmic material for violin 1 introduction.

Top row marks a twelve-quaver sequence of the values for *M*=12

Each *crescendos* from *pianissimo* at 0 to *forte* at its midpoint, to *pianissimo*. The dynamic at each value of the sieve is taken.

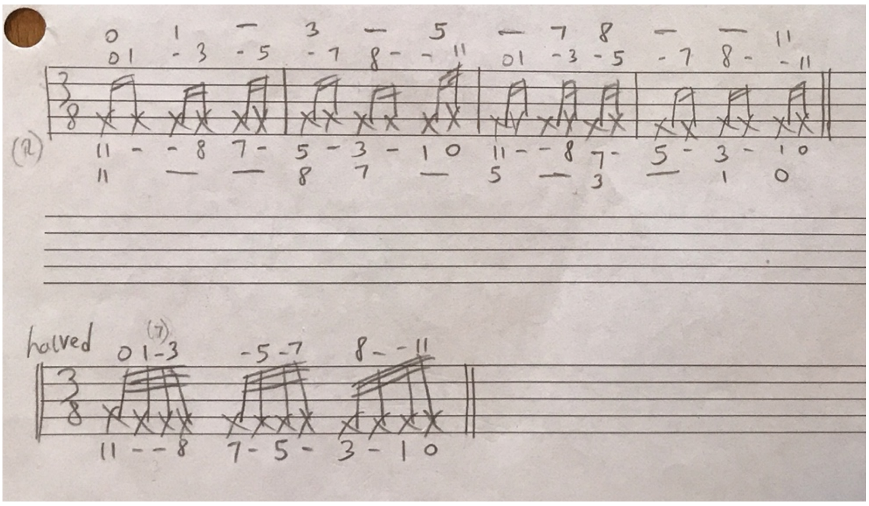
Bottom row marks a sequence of the values for *M*=24 cycled into a twenty-one-quaver phrase



character of accompanist with sustained notes, which it is to embody more fully in the proceeding section.

The onset of the first melodic passage at b. 111 presents a motif that will be transformed throughout the remainder of the piece. This first iteration is made up of *M*=12 in its unaltered and retrograde forms, as well as the halved and doubled note-value forms of each. Example 2 shows the method used to construct this.

Example 2: Construction of first melodic section (bb.111–118).



Numbers show temporal location of each sieve beat. Prime forms of *M*=12 form the top row, retrograde forms bottom.

Notes are divided amongst two instruments to form melodic passage. Where there are more notes than it is possible to distribute, a filtering process determines which notes are discarded.

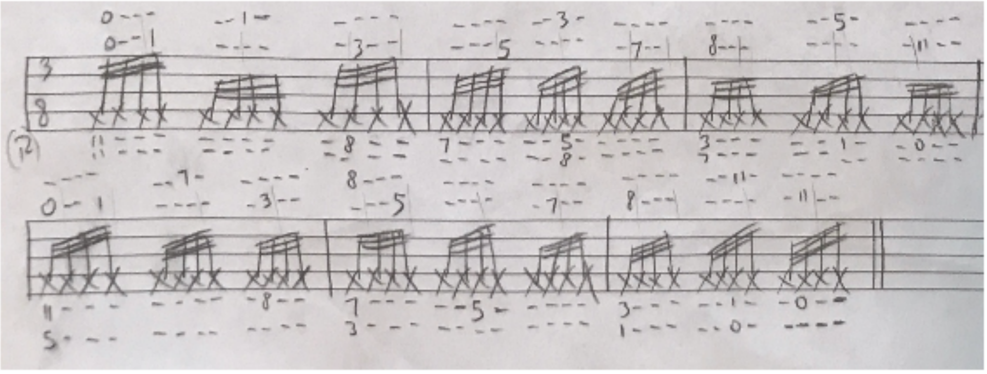
The transitionary passage encompassing bb. 119–130 contains *M*=12 in quadrupled note values in violin 2. Following this final interplay of material found in the first melodic section, *M*=12 appears multiplied by 1.5 in bb. 131–142. The addition of this uneven multiplication, counterpointed by *M*=12 in even multiples in violin 2, increases the number of dissonances between parts, and the rhythmic discrepancy between these transformations contributes to the two-bar ‘tag’ found in the second iteration of the melodic section at bb. 147–148 (Example 3).

The central part of the piece is the passage between melodic sections two and three (b. 155–196). With the introduction of sieves with *M* values of 21, new pitches are introduced for the first time:

That this sieve has an *M* value greater than the 12 available pitches results in a scale that effectively runs past the octave. The ‘octave’ in this sense may be thought of as E-natural, or the point at which the sieve repeats. The melodic passage following this passage was composed in advance:

The transition passage therefore has a ‘target’. The colouring in Example 4 show the notes to be included in the melodic section. Applying another filtering process, non-coloured notes are removed from what is initially a layering of all sieve material with each repetition, such that fragments of the melodic section begin to emerge from the texture over time. As melody is hocketed between instruments, there is a degree of aleatoricism built into the process regarding how and when these fragments emerge.

Example 3: Construction of second melodic section (bb. 143–148).

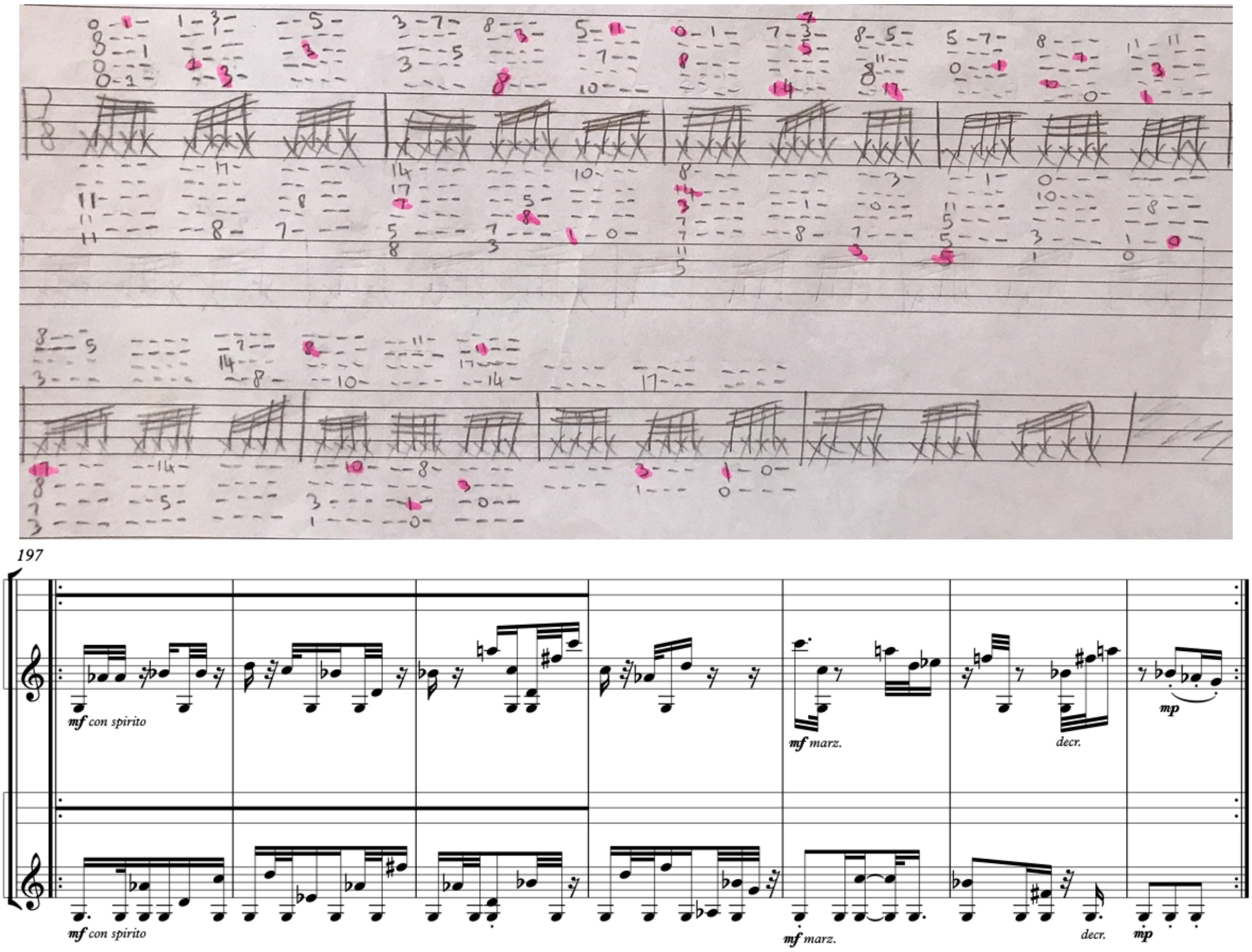


Melodic section is now six bars long due to inclusion of *M*=12x1.5 (*M*=18) and its transformations

Notes found in the last two bars are the product of *M*=12x1.5 (*M*=18) and its transformations

Example 4: Pitch and rhythmic material heard at the onset of final transitional passage (bb. 155–196). Non-coloured notes are filtered over the course of the section.

Example 5: Construction of third melodic section (bb.197–203).



Owing to the inclusion of *M*=21, the third melodic section spans seven bars. After a recapitulation of material from the piece's opening, the final gesture of the piece is the squaring of the seven-bar melody with the sieve of *M*=24, which lasts eight bars and therefore synthesises all of the material extraneous to *M*=12. A coda employs this melodic material more freely than in preceding sections, and violin 2 quotes the first melodic area directly. The piece concludes with a notated deceleration, something I became interested around the time and would later seek to formalise (bb. 273–278). The title, drawn from 'tetravigesimal', the name for the base-24 number system, is a so-called ‘kangaroo word’, containing within it the word ‘trade’ which I found appropriate to the hocketing-heavy nature of the piece.

I found sieve theory to be musically rewarding, and was intrigued by the rhythmic variety that sieves can produce. Further investigation might look to the use of cyclic transposition of sieves (changing the order of spacing)—in effect creating ‘modes’—and more complex transformations. However it was the textural, filtering passages between melodic areas that most caught my interest; the effect of being ‘in between’ states here was something I sought to investigate further, and tool shape in my later work.

Bibliography

Exarchos, Dimitrios. "Iannis Xenakis and Sieve Theory: An Analysis of the Late Music (1984-1993)". Goldsmiths, University of London, 2007.